M. Sc. III SEMESTER

Semester Examination 2021
Subject: Mathematics
Paper Code: MMA-E302
Paper Name: GRAPH THEORY

TIME: 3 Hrs **MAX. MARKS: 70**
Min. Pass % : 40

Note: This question paper is divided into two sections ***A*** and ***B***. Attempt all sections as per instructions.

Section-A (Short Answer Type Questions)

Note: Answer any **FIVE** questions in about 150 words each. Each question carries **SIX marks**.

1. Prove that a tree with n vertices have n-1 edges.
2. When two graphs are are said to be isomorphic? Show that the two graphs need not be isomorphic even when they have the same order and same size.
3. Prove that if a connected graph G is decomposed into two subgraphs g1 and g2 there must be one vertex common between g1 and g2.
4. Sketch a graph G that has the following vectors in its circuit subspace:(0,1,1,1,1,0,0,1),(0,1,1,1,0,1,0),(0,1,0,0,1,0),(0,1,0,0,1,0,1),(1,0,1,0,1,1,0,1),(1,0,1,0,0,0,1,0),(1,0,0,1,1,1,1,0),(1,0,0,1,0,0,1).
5. Show that if a graph G have one and only one path between every pair of vertices , then G is a tree.
6. Explain Kuratowski's two non-planar graphs. Show that the complete graph of five vertices is nonplanar.
7. Define the thickness and crossing number of a graph. Find the thickness and crossing number of the complete graph with n vertices, where $n \leq 8$.
8. Define a circuit vector and a cut set vector of a connected graph. Prove that a circuit vector and a cut set vector are orthogonal to each other w.r.t. mod 2 arithmetic.
9. Show that a connected graph is an Euler graph if it can be decomposed into edge disjoint circuits.
10. Prove that in a non-trivial tree T there are atleast two pendant vertices.

Section-B (Long Answer Type Questions)

NOTE: Answer any **FOUR** questions in detail. Each question carries **TEN marks**.

1. Show that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.
2. Consider a tree T with 3 vertices of degree 2, 4 vertices of degree 3 and 3 vertices of degree 4. Calculate the number of pendant vertices.
3. Define network and separable graph. Prove that the vertex connectivity of a connected graph G cannot exceed the degree of the vertex v, where v has the smallest degree in G.
4. Explain Kuratowski's two non planar graphs. Show that the complete graph of five vertices is non planar.
5. Prove that e-n+2= r. (Euler's formula)
6. Discuss Incidence matrix, circuit matrix, Path matrix, Cut set matrix with examples.
7. Describe an algorithm to detect the planarity of a graph. Detect the planarity of $K_{3,3}$.
8. Explain Kruskal's and Dijakstra Algorithms.

*****~*****~*****