SEMESTER EXAMINATION-2021
CLASS – M.SC. SUBJECT - MATHEMATICS
PAPER CODE: MMA-C302
PAPER TITLE: MEASURE THEORY

Time: 3 hour **Max. Marks: 70**
Min. Pass: 40%

Note: Question Paper is divided into two sections: **A and B.** Attempt both the sections as per given instructions.

SECTION-A (SHORT ANSWER TYPE QUESTIONS)

Instructions: Answer any five questions Each question carries six marks. (5 X 6 = 30 Marks)

Question-1: Define cardinal number of a set and Let P and Q be any two sets, then show that
(a) card P + card Q is unique
(b) card P. card Q is unique

Question-2: Define Countable set with an example and Find the power of an aggregate of numbers given by $\frac{M}{2^m}$, M and m being positive integers.

Question-3: Define Lebesgue Outer Measure. Prove that If $< E_1, E_2, \ldots >$ is a monotonically decreasing sequence of measurable sets such that $m(E_1) < \infty$ and $E = \bigcap_{k=1}^{\infty} E_k$,
then $m(E) = \lim_{n\to\infty} m(E_n)$

Question-4: (i) Define with an example:
(a) Borel set
(b) F_σ and G_δ set
(c) Non-measurable set
(ii) Show that a Borel measurable set is Lebesgue measurable.

Question-5: Let f and g are measurable functions define over a measurable set E. show that f+g , f –g , fg and f/g are measurable functions over E.

Question-6: Define characteristic function and show that the characteristic function of E is measurable if and only if E is measurable set.

Question-7: If f is a bounded function defined on[a, b] and f is R-integrable on [a, b] than f is also L-integrable on [a, b] and $L\int_a^b f = R\int_a^b f$

Question-8: If f is a bounded function, Lebesgue integral on a measurable sub-set of [a, b], then $|f|$ *is also* $L-$ *integrable on E and* $\left|\int_E f\right| \leq \int_E |f|$

Question-9: Define function of bounded variation and prove that a monotonic function on [a, b] is of bounded variation.

Question-10: Define $L^p -$ *space and norm of an element of* $L^p -$ *space and let* $f \epsilon L^p[a,b], g \epsilon L^p[a,b]$; *then* $f + g \epsilon L^p[a,b]$

SECTION-B (LONG ANSWER TYPE QUESTIONS)

Instructions: Answer any FOUR questions in detail. Each question carries 10 marks. (4 X 10 = 40 Marks)

Question-1: Define Cantor set and construct Cantor ternary set and prove that its measure is zero.

Question-2: The Outer measure of an interval is equal to its length.

Question-3: State and Prove Egoroff's Theorem.

Question-4: State and prove Lebesgue Dominated Convergence Theorem.

Question-5: Let f be a bounded function defined on [a,b]. Then the function f is l-integrable, iff for each ε >0, however small, there exists a measurable partition P of [a, b] such that U[f ; P] – L[f ; P] < ε

Question-6: State and prove Cauchy-Schwarz inequality.

Question-7: Define absolutely continuous function and prove that if a function f is absolutely continuous in an interval[a, b] and if f '(x) = 0 a.e. in [a, b], then f is constant.

Question-8: (i) the union of finite numbers of measurable sets is also measurable i.e, Lebesque measure is finitely additive.
(ii) The close interval A = [0, 1] is uncountable.

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