SEMESTER EXAMINATION-2021

CLASS:M. Sc. I Semester **SUBJECT:MATHEMATICS**

PAPER CODE and NAME: MMA-C113(Advanced Real Analysis)

Time: 3 hour **Max. Marks: 70**

Min. Pass: 40%

Note: Question Paper is divided into two sections: **A and B.** Attempt both the sections as per given instructions.

SECTION-A (SHORT ANSWER TYPE QUESTIONS)

Instructions: Answer any FIVE questions in about 150 words each. Each question carries **6** marks. **(5 X 6 = 30 Marks)**

Question-1: Test the uniform convergence for the sequence $\{f_n\}$ where $f_n(x) = e^{-nx}$ for $x \geq 0$.

Question-2: Show that a set is infinite iff it contains a denumerable subset.

Question-3: Prove that every set E having outer measure zero, is measurable. Hence or otherwise, show that every subset of E is measurable.

Question-4: Prove that a measurable function f is integrable over E if and only if $|f|$ is integrable over E .

Question-5: Test the uniform convergence for the series $\sum_{n=1}^{\infty} \frac{x}{n(n+1)}$ in $]0, k[$ and $]0, \infty[$.

Question-6: Prove that a continuous function defined on a measurable set is measurable.

Question-7: If f and g are measurable functions on a common domain E , then show that each of the sets $\{x \in E: f(x) \leq g(x)\}$ and $\{x \in E: f(x) = g(x)\}$ are measurable.

Question-8: Prove that almost uniform convergence implies convergence in measure.

Question-9: Prove that a Riemann integrable function defined on $[a, b]$ is measurable.

Question-10: If f is an integrable function such that $f = 0$, then show that $\int f = 0$

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SECTION-B (LONG ANSWER TYPE QUESTIONS)

Instructions: Answer any FOUR questions in detail. Each question carries **10** marks. **(4 X 10 = 40 Marks)**

Question-11: If a sequence $\{f_n\}$ converges uniformly to f on $[a,b]$ and each function f_n is integrable on $[a,b]$, then show that f is integrable on $[a,b]$and the sequence $\left\{\int_a^b f_n\,dx\right\}$ converges uniformly to $\int_a^b f\,dx$.

Question-12: If a sequence $\{f_n\}$ converges in measure to a function f on E, then show that there exists a subsequence $\{f_{n_k}\}$ of $\{f_n\}$which converges to f *a.e.* on E.

Question-13: Prove that every Borel set in R is measurable.

Question-14: State and prove Fatou's lemma.

Question-15: Test the uniform convergence and continuity of the following sequences:

(a). $\{x^n\}$ where $x \in [0,1]$ (b). $\left\{\frac{1}{1+nx}\right\}$ where $x \in [0,1]$.

Question-16: Let $\{E_i\}$ be an infinite increasing sequence of sets, then show that $m^*(\bigcup_{i=1}^{\infty} E_i) = \lim_{n\to\infty} m^*(E_n)$.

Question-17: If f is a measurable function defined on E, then show that for each $\epsilon > 0, \exists$ a closed set $F \subset E$ with $m(F - E) < \epsilon$ such that f is continuous on F.

Question-18: Let $\{u_n\}$ be a sequence of integrable functions on E such that $\sum_{n=1}^{\infty} u_n$ converges *a.e.* on E. Let g be a function which is integrable on E and satisfy $|\sum_{i=1}^{n} u_i| \le g$ *a.e.* on E for each n. Then show that $\sum_{n=1}^{\infty} u_n$ is integrable on E and$\int_E \sum_{n=1}^{\infty} u_n = \sum_{n=1}^{\infty} \int_E u_n$.

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