SEMESTER EXAMINATION-2021
CLASS – M.SC. SUBJECT - MATHEMATICS
PAPER CODE: MMA-C111
PAPER TITLE: ORDINARY DIFFERENTIAL EQUATIONS

Time: 3 hour **Max. Marks: 70**
Min. Pass: 40%

Note: Question Paper is divided into two sections: **A and B.** Attempt both the sections as per given instructions.

SECTION-A (SHORT ANSWER TYPE QUESTIONS)

Instructions: Answer any five questions Each question carries six marks. (5 X 6 = 30 Marks)

Question-1: State Convolution Theorem and hence evaluate $L^{-1}(\frac{P}{(P^2+1)(P^2+4)})$.

Question-2: Find the Laplace transform of $\int_0^{\infty} e^{-st}\, t^3\, cost\, dt$.

Question-3: Prove that

$$\int_{-1}^{1} x^2\, P_{n-1}(x) P_{n+1}(x) dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}.$$

Question-4: Prove that

$$(2n+1)(x^2-1)P_n' = n(n+1)(P_{n+1} - P_{n-1}).$$

Question-5: Prove that

$$J_{\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left[\left(\frac{3-x^2}{x^2}\right) sinx - \frac{3cosx}{x}\right].$$

Question-6: Prove that

(i) $4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$

(ii) $J_0'(x) = -J_1(x)$.

Question-7: Prove that

$$2xH_n(x) = 2xH_{n-1}(x) + H_{n+1}(x).$$

Question-8: State and prove Generating function for Hermite Polynomial $H_n(x)$.

Question-9: Show that $(n+1)L_{n+1}(x) = (2n+1-x)L_n(x) - nL_{n-1}(x)$.

Question-10: Find the third approximation of the equation $\frac{dy}{dx} = z,\ \frac{dz}{dx} = x^3(y+z)$ by Picard's method where y = 1, z = ½, when x = 0

SECTION-B (LONG ANSWER TYPE QUESTIONS)

Instructions: Answer any FOUR questions in detail. Each question carries 10 marks. (4 X 10 = 40 Marks)

Question-1: Solve $y''(t) - ty' + y = 1, if\ y(0) = 1, y'(0) = 2.$

Question-2: Solve in series the differential equation

$$x^2 \frac{d^2y}{dx^2} + 5x \frac{dy}{dx} + x^2 y = 0.$$

Question-3: If α and β are the roots of $J_n(x) = 0$, then

$$\int_0^1 x J_n(\alpha x) J_n(\beta x) dx = \left\{ \begin{matrix} 0, & when\ \alpha \neq \beta \\ \frac{1}{2} J_{n+1}^2(\alpha), & when\ \alpha = \beta \end{matrix} \right\}.$$

Question-4: Show that $(1-2xz+z^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)\, z^n$

Question-5: State and prove Rodrigue's formula for Hermite Polynomials

Question-6: State Laguerre's Differential Equation and solve it.

Question-7: Solve $(x+2)\frac{d^2y}{dx^2} - (2x+5)\frac{dy}{dx} + 2y = (x+1)e^x$

Question-8: Solve by method of variation of parameters

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = x^2 e^x.$$

Paper Code: MMA-C111