SEMESTER EXAMINATION-2021

CLASS:B. Sc. III Semester **SUBJECT:MATHEMATICS**

PAPER CODE and NAME: BMA-S301(Logic and Sets)

Time: 3 hour **Max. Marks: 70**

Min. Pass: 40%

Note: Question Paper is divided into two sections: **A and B.** Attempt both the sections as per given instructions.

SECTION-A (SHORT ANSWER TYPE QUESTIONS)

Instructions: Answer any FIVE questions in about 150 words each. Each question carries **6** marks. **(5 X 6 = 30 Marks)**

Question-1: With usual notations, construct the truth table of $(p \wedge q) \vee (q \wedge r) \vee (r \wedge p)$.

Question-2: Let $A(x)$, $B(x)$ and $C(x)$, respectively, be stand for " x has a white color " , " x is a polar bear " and " x is found in cold region " over the universe of animals. Then translate the following into theoretical propositions (in sentences form):

(a). $\exists\, x\left(B(x) \wedge \sim A(x)\right)$ (b). $(\exists\, x)\left(\sim C(x)\right)$ (c). $(\forall\, x)\left(B(x) \wedge C(x) \rightarrow A(x)\right)$

Question-3: Let p, q and r, respectively, be stand for "He is Coward." , "He is lazy." and "He is rich." . Then write the compound symbolic form corresponding to the following theoretical propositions:

(a). He is either coward or poor.

(b). He is rich or else he is both coward and lazy.

(c). It is false that he is coward but not lazy.

(d). He is coward or lazy but not rich.

(e). It is false that he is coward or lazy but not rich.

(f). He is neither coward nor lazy.

Question-4: With usual notations, construct the truth table of $x \uparrow y \uparrow z$.

Question-5: With usual notations, prove that the following propositions are equivalent to $u \rightarrow v$.

(a). $\sim (u \wedge \sim v)$ (b). $\sim u \vee v$ (c). $\sim v \rightarrow \sim u$

Question-6: State and prove commutative laws for sets.

Question-7: If $A=\{1,2,4,5\}$, $B=\{a,b,c,f\}$ and $C=\{a,5\}$ then compute $(A\cup C)$ and $(A\cup C)\times B$.

Question-8: Write the following sets in builder form:

(a). $A=\{13,26,39,52,....\}$ (b). $B=\{1,16,81,256,....\}$

(c). $C=\{$ Uttar Pradesh, Uttarakhand, Delhi, Haryana, ..., Bihar $\}$

Question-9: For the sets of identity $P\cap Q=P\cap R$, is it necessary that $Q=R$? Justify your answer with suitable example.

Question-10: Prove that a relation R on a set A is symmetric iff $R=R^{-1}$.

SECTION-B (LONG ANSWER TYPE QUESTIONS)

Instructions: Answer any FOUR questions in detail. Each question carries **10** marks. **(4 X 10 = 40 Marks)**

Question-11: State and verify distributive laws for conjunction and disjunction.

Question-12: Discuss the nature (as tautology, contingency or contradiction) of the following propositions:

(a). $A\vee\sim(A\wedge B)$ (b). $A\rightarrow A\wedge(A\vee B)$

(c). $A\rightarrow(B\rightarrow A)$ (d). $(A\rightarrow B)\rightarrow(A\wedge B)$

Question-13: Without using truth tables show that the following arguments are valid:

(a). $P\rightarrow\sim Q,\ R\rightarrow Q, R\ |-\ \sim P$

(b). $(P\wedge Q)\vee(R\rightarrow S),\ T\rightarrow R,\ \sim(P\wedge Q)\ |-\ T\rightarrow S$

Question-14: Test the truthness of the following propositions:

(a). $\forall n\in N,\ n+4>3$ (b). $\forall n\in N,\ n+2>8$ (c). $\exists\, n\in N,\ n+4<7$

(d). $\exists\, n\in N,\ n+6<4$ (e). $\exists\, n\in N,\ n^2-2n+5=0$

Question-15: State and prove associative laws for sets under union and intersection.

Question-16: For three sets A, B and C, prove that $A\times(B\cap C)=(A\times B)\cap(A\times C)$.

Question-17: Let $U=\{1,2,3,4,5,6,7,8,9\}$, $A=\{1,4,9\}$, $P=\{x\in U : x \text{ is a perfect square}\}$, $R=\{1,2,3,5,7,9\}$, $D=\{2,3,5,7\}$, $N=\{x\in U : x \text{ is a prime number}\}$ and ϕ =empty set . Then determine:

(a). which sets are subsets of others.

(b). which sets are proper subsets of others.

(c). pair of sets which are disjoint.

(d). pair of sets which are comparable.

(e). pair of sets which are incomparable.

Question-18: Let U = universal relation , $R=\{(1,1),(1,2),(1,3),(3,3)\}$, $S=\{(1,1),(1,2),(2,1),(2,2),(3,3)\}$, $T=\{(1,1),(1,2),(2,2),(3,3)\}$ and ϕ =empty relation be the relations on $A=\{1,2,3,4\}$. Then determine whether or not each of the above relations on A is :

(a). reflexive (b). symmetric

(c). transitive (d). anti-symmetric

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