B. Sc. I SEMESTER

Semester Examination 2021
Subject: Mathematics
Paper Code: BMA-C101
Paper Name: DIFFERENTIAL CALCULUS

TIME: 3 Hrs **MAX. MARKS: 70**
Min. Pass % : 40

Note: This question paper is divided into two sections ***A*** and ***B***. Attempt all sections as per instructions.

Section-A (Short Answer Type Questions)

Note: Answer any **FIVE** questions in about 150 words each. Each question carries **SIX marks**.

1. Test the following function for continuity at x =0: $f(x) = \frac{1}{1-e^{-1/x}}\ x \neq 0,\ f(x) = 0, at\ x = 0.$
2. If $u = e^{xyz}$ show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2)e^{xyz}.$
3. State and prove Euler's theorem on homogeneous functions.
4. Find the nth differential coefficient of $\tan^{-1}\left(\frac{2x}{1-x^2}\right).$
5. Prove that $(1+x) < e^x < 1 + xe^x, \forall\ x > 0.$
6. Show that the function $f(x) = x|x|$ is differentiable at x=0.
7. Expand $\log \sin(x+h)$ in powers of h by Taylor's theorem.
8. Show that the spiral $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ intersect orthogonally.
9. Find all the asymptotes of the curve $y^3 - xy^2 - x^2 y + x^3 + x^2 - y^2 - 1 = 0.$
10. Show by using epsilon-delta definition of limit that $\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0.$

Section-B (Long Answer Type Questions)

NOTE: Answer any **FOUR** questions in detail. Each question carries **TEN marks**.

1. Show that continuity is the necessary condition for the existence of a finite derivative.
2. Let $f(x) = x\dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}, x \neq 0$ and $f(0) = 0.$ Show that the function $f(x)$ is continuous at x = 0 but not differentiable at x = 0.
3. Trace the curve $y^2(2a - x) = x^2.$
4. If $y = e^{\tan^{-1} x}$, prove that $(1 + x^2)y_{n+2} + [(2n+1)x - 1]y_{n+1} + n(n+1)y_n = 0.$
5. State and prove Rolle's theorem also verify the theorem for the function $f(x) = |x|$. on the interval [-1, 1].
6. Show that the pedal equation of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1,$ is $\dfrac{1}{p^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2} - \dfrac{r^2}{a^2b^2}.$
7. State and prove Maclaurin's theorem. Apply Maclaurin's theorem to prove that $\log\sec x = \dfrac{1}{2}x^2 + \dfrac{1}{12}x^4 + \dfrac{1}{45}x^6 + ...$
8. If $x^x y^y z^z = c.$ prove that at $x = y = z, \dfrac{\partial^2 z}{\partial x \partial y} = -[x \log ex]^{-1}.$

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