B. Sc. III SEMESTER

Semester Examination 2021
Subject: Mathematics
Paper Code: BMA-C301
Paper Name: REAL ANALYSIS

TIME: 3 Hrs **MAX. MARKS: 70**
Min. Pass % : 40

Note: This question paper is divided into two sections ***A*** and ***B***. Attempt all sections as per instructions.

Section-A (Short Answer Type Questions)

Note: Answer any **FIVE** questions in about 150 words each. Each question carries **SIX marks**.

1. Prove that every subset of a countable set is countable?
2. Find the infimum and supremum of the set $S = \left\{ 1 + \frac{(-1)^n}{10n} \text{ s.t. } n \in N \right\}$.
3. Discuss the convergence of the sequence $\left\langle \frac{1}{3^n} \right\rangle$.
4. Show that the constant sequence $< f_n >$, where $f_n = c \;\forall\; n \in N$, converges to c.
5. Test the given series for convergence $\sqrt{n^4+1} - \sqrt{n^4-1}$.
6. Define Archimedean property of R and, supremum and infimum of a set?
7. Suppose $\sum_n a_n$ converges, then prove that $\sum_n \frac{a_n}{n^x}$ converges uniformly in [0, 1].
8. Find the radius of convergence of the power series $\sum_n \frac{(\mathrm{x}-1)^n}{3^n (n+1)^3}$.
9. State and prove Bolzano Weirstrass theorem.
10. Show that every convergent sequence is bounded.

Section-B (Long Answer Type Questions)

NOTE: Answer any **FOUR** questions in detail. Each question carries **TEN marks**.

1. Show that the sequence $\langle s_n \rangle$ defined by the formula $s_1 = 1, s_{n+1} = \sqrt{3s_n}$ converges to3.

2. Test the convergence of $1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + ... for\ p > 0$.

3. State and prove Cauchy's first theorem on limits.

4. Test for uniform convergence, the sequence $\{ f_n \}$, where

   $f_n = \frac{nx}{1+n^2x^2}\ ,\ for\ all\ real\ x.$

5. Prove that the open interval (0, 1) is an uncountable set?

6. State and prove the Cauchy convergence criterion of limit.

7. Define conditionally convergent and absolutely convergent series. Show that the series

   $\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + ...$ is conditionally convergent.

8. Define monotonic sequence. Show that every bounded monotonically increasing sequence converges.